Statistical fluctuations and frequency distribution of

counting rates (Item No.: P1314500)

Curricular Relevance



Task and equipment

Introduction

Radioactive decays are statistical processes. For this reason, the impulse rates Z registered under the same experimental conditions vary around a mean value Z. Each individual measurement carries a statistical uncertainty s, which reduces by increasing the total number of registered impulses N. According to the formula:

$s = \pm \sqrt{N}$

The formula is applicable for these statistical errors. For a sufficiently large number of individual measurements 68% of the impulses lie in the range $\pm s$, and the remaining can show bigger variations from the mean value. The relative statistical error or the standard deviation is:

$$\rho = \pm \frac{1}{\sqrt{N}}$$

For example, if the mean value of the number of impulses is 100 impulses, then it can be expected, that 68% of the values would be in the range between 90 and 110 impulses. The statistical error of the individual measurement is $s = \pm 10$ impulses and the standard deviation is $\rho = \pm 10\%$.

If the measurement time is increased to register about 400 impulses, then although the absolute statistical error rises to $s = \pm 20$ impulses, the standard deviation reduces to $\rho = \pm 5\%$. Hence, by selection of the measurement time, one can bring the statistical error below the desired value. Since a large number of similar measurements is necessary, it is recommended to involve the students in the registration and evaluation of the measurements. Preparation of work sheets with the necessary tables and coordinate systems is recommended for evaluation.

Task

Student's Sheet

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Equipment

Position No.	Material	Order No.	Quantity
1	Radioactive sources, set	09047-50	1
2	Geiger-Müller-Counter	13606-99	1
3	Demo Physics board with stand	02150-00	1
4	Geiger-Mueller counter tube, 15 mm (type B)	09005-00	1
5	Clamp on holder	02164-00	1
6	Counter tube holder on fixating magnet	09201-00	1
7	Source holder on fixing magnet	09202-00	1
8	Support clamp for small case	02043-10	1
9	Support rod,stainl.steel, 100mm	02030-00	1

Set-up and procedure

Set-up

Fig. 1 shows the set-up of the experiment. Remove the protective cap from the counter tube and select the measurement time "Auto/10 s" at the counter.

Set the distance between the Strontium-90 radiation source and the counter tube in such a way, that the impulse rate is about 300-600 Imp/10 s.



Procedure

Determine the impulse rates of at least 50 measurements and enter them in the prepared table.

After concluding the measurements place the protective cap back on the counter tube and place the radiation source back in the storage container.



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Results and evaluation

Results

lfd Nr	Z	lfd Nr		lfd Nr	Z	lfd Nr	Z
	$\overline{Imp/10~s}$	LIG. MI.	$\overline{Imp/10~s}$	LIU. NI.	$\overline{Imp/10~s}$	LIG. NI.	$\overline{Imp/10~s}$
1	515	16	498	31	460	46	544
2	482	17	507	32	553	47	490
3	485	18	486	33	496	48	529
4	494	19	512	34	513	49	506
5	486	20	476	35	502	50	499
6	457	21	534	36	478		
7	487	22	478	37	468		
8	500	23	548	38	513		
9	497	24	518	39	542		
10	535	25	515	40	488		
11	537	26	485	41	522		
12	512	27	491	42	507		
13	486	28	497	43	507		
14	513	29	509	44	505		
15	513	30	528	45	534		

Evaluation

The mean value of all the counting rates is

 $Z\,$ = 504.7 Imp/10 s \approx 505 Imp/10 s

These measurement values in this example have the largest deviation from the mean value $Z_6 = 457 \text{ Imp}/10 \text{ s} = Z_{min}$ and $Z_{32} = 553 \text{ Imp}/10 \text{ s} = Z_{max}$. The maximum deviation from the mean value is $\pm 48 \text{ Imp}/10 \text{ s}$.

The difference between the smallest and the largest measurement value is Z_{max} – Z_{min} = ΔZ = 96 Imp/10 s.

For studying the distribution of the measurement values around the mean value, the total range of the impulse rates from Zmin to Zmax is divided into sub-ranges. The number of sub ranges should not be too high as due to the already low number of measurement values, then the number of values in the individual sub ranges will also become low. If the complete range is divided into 6 ranges, then one sub range would include

$$\frac{\Delta Z}{6} \approx \frac{16 \ Imp}{10 \ s}.$$

This results in the following ranges:

Table 2		
No. of range	Impulse rates	
1	457-473	
2	474-490	
3	491-507	
4	508-524	
5	525-541	
6	542-558	

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Table 3		
No. of	Count of measured	
range	values	
1	3	
2	12	
3	14	
4	11	
5	6	
6	4	

Fig. 2 shows the frequency distribution of the measured values. It can be seen here, that the values appear with the highest frequency near the mean value. A theoretical (envelope / wrap-up curve) drawn over a bar graph has the form of a bell-curve; in case of a large number of measurement values, this distribution curve describes the theoretically expected distribution of the measurement values.



Within the mean statistical error of the mean value i.e. in the range

 $\overline{Z} \pm \sqrt{Z} = (505 \pm 23) \text{ Imp/10 s} = (482-528) \text{ Imp/10 s},$ there are 34 values, which is 68% of the total. This conformation/agreement of the result with the expected one in the case of any number of measurement values in coincidental.